Maths Mastery

At the depth of the mastery approach to the teaching of mathematics is the belief that **all children have the potential to succeed.** They should have access to the same curriculum content and, rather than being extended with new learning, they should **deepen their conceptual understanding by tackling challenging and varied problems.** Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding through the use of concrete materials and pictorial representations. This policy outlines the different calculation strategies that should be taught and used in EYFS through to Year 6 in line with the requirements of the 2014 Primary National Curriculum.

Mathematical Language

The 2014 Primary National Curriculum is explicit in articulating the importance of children using the correct mathematical language as part of their learning. Indeed, in certain year groups, the non-statutory guidance highlights the requirements for children to extend their language around certain concepts. It is therefore essential that teaching the strategies outlined in this policy is accompanied by the use of appropriate and precise mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers only accepting what is correct. The agreed list of terminology is above each mathematical operation in this policy.

> The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof.
>
> 2014 Maths Programme of Study

How to use the policy

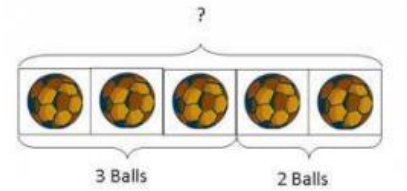
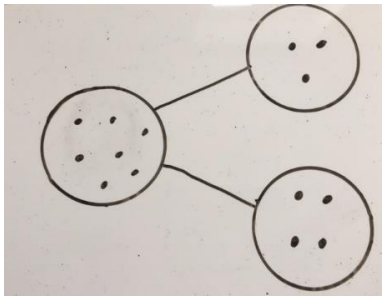
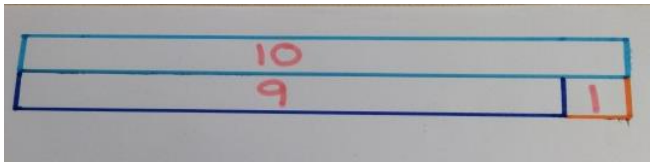
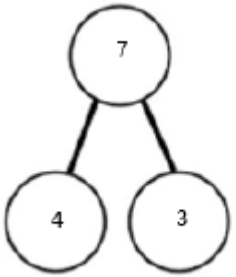
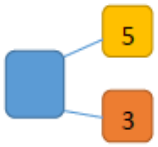
This policy is a guide for all teaching and support staff. It is purposefully set out as a progression of mathematical skills and not into year group phases to encourage a flexible approach to teaching and learning. It is expected that teachers will use their professional judgement as to when consolidation of existing skills is required or if to move onto the next concept. However, the focus always **must remain on breadth and depth rather than accelerating through concepts.** Children should not be extended with new learning before they are ready, they should deepen their conceptual understanding by tackling challenging and varied problems.

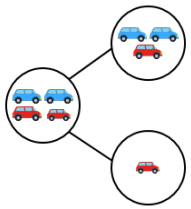
For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The main concrete materials to be used within all year groups are Base 10, PV counters and Cuisenaire rods. The principle of the concrete-pictorial-abstract (CPA) approach (make it, draw, write it) is for children to have a true understanding by mastering all these three phrases within each mathematical concept.

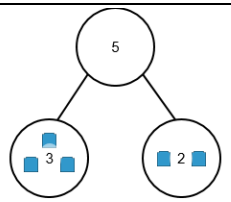
Calculation Policy: Guidance Addition and Subtraction

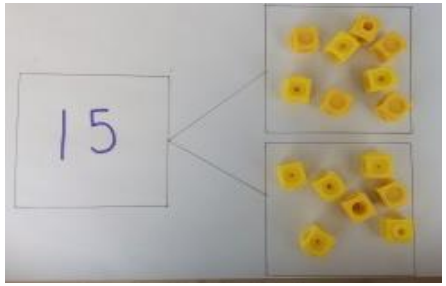
	EYFS	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model Start with the bigger number and count on Regrouping to make 5 using the five frame	Combining two parts to make a whole: part whole model Start with the larger number and count on Regrouping to make 10 using the ten frame	Adding 3 single digit numbers Use of base 10 to combine two numbers Use of a blank number line	Column method – regrouping Using PV counters (up to 3 digits)	Column method – regrouping (up to 4 digits)	Column method – regrouping PV with decimals	Column method with regrouping Abstract methods
Subtraction	Counting back Taking away ones Part whole model Making 5 using the five frame	Counting back Taking away ones Find the difference Part whole model Make 10 using the 10 frame	-Counting back -Find the difference -Part whole model -Blank number line -Make 10 -Use of base 10	Column method with regrouping (up to 3 digits using PV counters	Column method with regrouping (up to 4-digits)	Column method with regrouping Abstract for whole numbers PV with decimals	Column method with regrouping Abstract methods

Calculation policy - Addition
Key Language: sum, total, parts and wholes, plus, add, addition, addend, total, altogether, score, more, is equal to, is the same as, exchange, inverse, make, increase, commutative

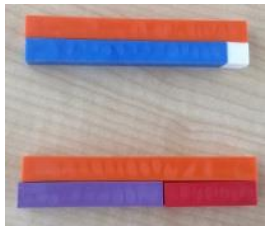
Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars).  Use cubes to add two numbers as a group or in a bar.   part part whole model 	   **Tens Writing frame** 	  Use the part-part whole diagram as shown above to move into the abstract. **4 + 3 = 7** **10= 6 + 4** **1 + 9 =10** **9 + 1 = 10** **10 – 1 = 9** **10 – 9 = 1**

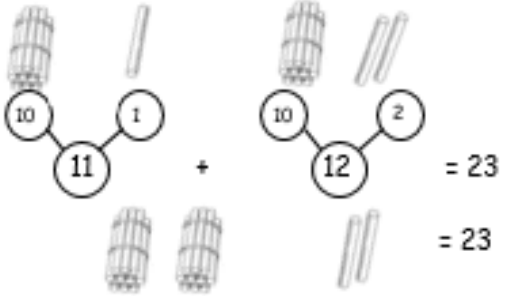




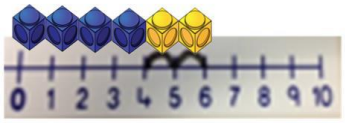


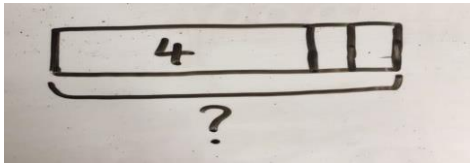
Link to addition – use the part whole model to explain the inverse between addition and subtraction.



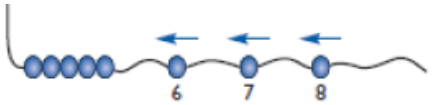


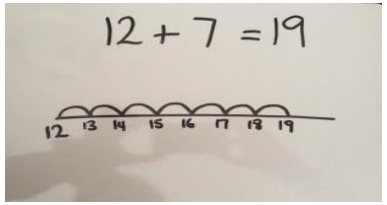
Counting on using number lines





Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.





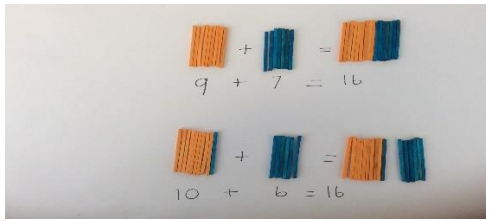
Start at the larger number on the number line and count on in ones or in one jump to find the answer.

The abstract number line:
What is 2 more than 4?
What is the sum of 2 and 4?
What is the total of 4 and 2?
4 + 2

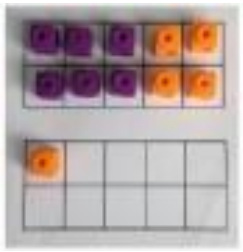
5 + 12 = 17

Place the larger number in your head and count on the smaller number to find your answer.

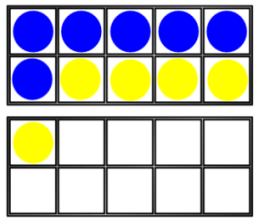
Regrouping to make 10. Continue to develop understanding of partitioning and place value.



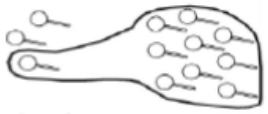
6 + 5 = 11



Use the tens frames to draw the counters



Use pictures or a number line. Regroup or partition the smaller number to make 10.



Children to develop an understanding of equality e.g.

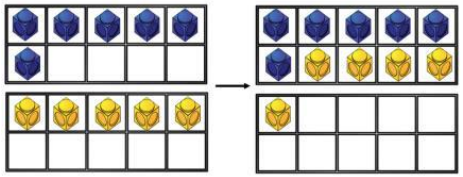
6 + □ = 11

6 + 5 = 5 + □

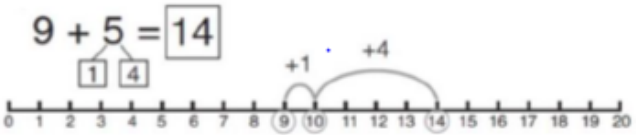
6 + 5 = □ + 4

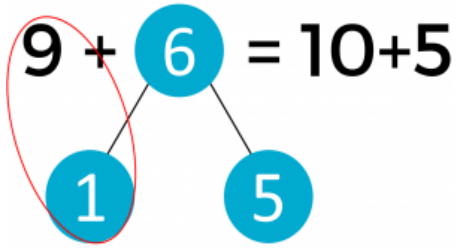
7 + 4= 11

If I am at seven, how many more do I need to make 10? How many more do I add on now?



Start with the bigger number and use the smaller number to make 10.





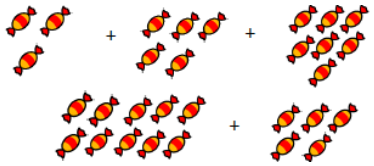
Adding three single digits.

4 + 7 + 6= 17

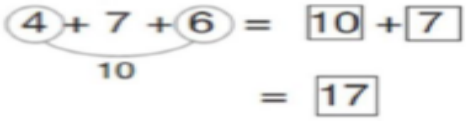
Put 4 and 6 together to make 10. Add on 7.

Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.

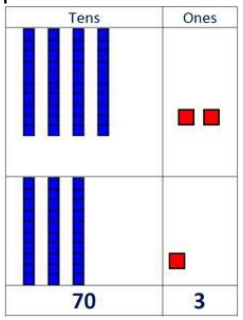
Add together three groups of objects. Draw a picture to recombine the groups to make 10.

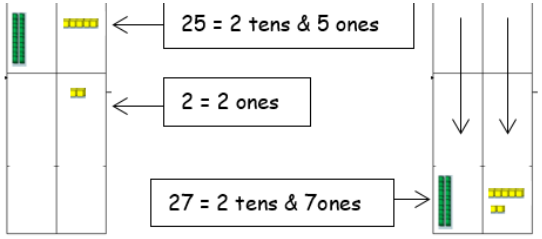


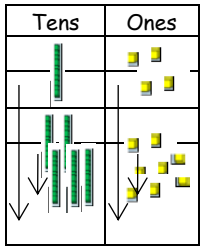
Combine the two numbers that make 10 and then add on the remainder.



TO + O and TO + TO using base 10. No regrouping
Continue to develop understanding of partitioning and place value.

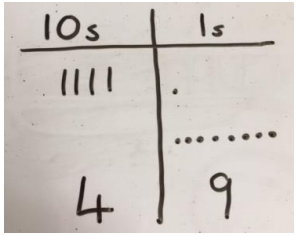




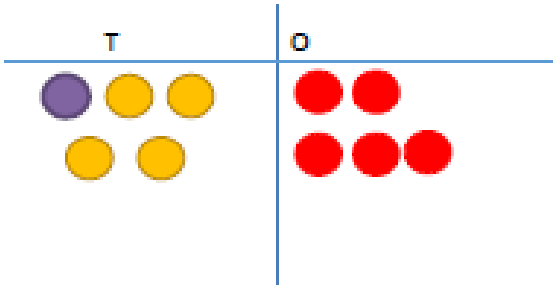


Add together the ones first then add the tens. Use the Base 10 first before moving onto place value counters.

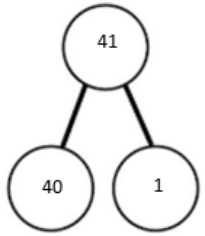
24 + 15=

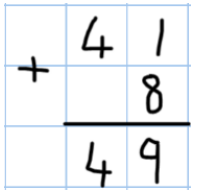


After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.



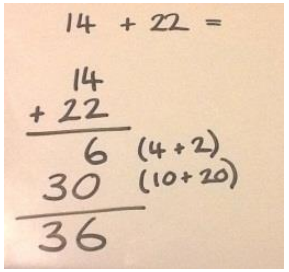
41 + 8 =
1 + 8 = 9 40 + 9 = 49





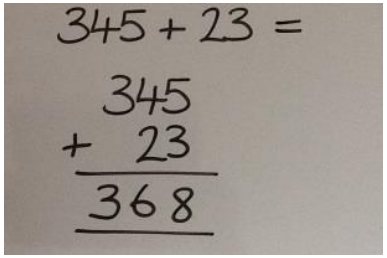
42 + 31 = 73

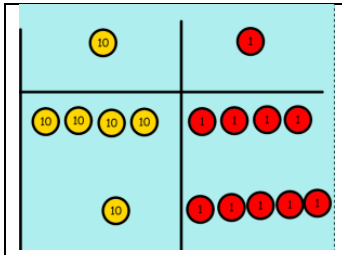
40 + 2
30 + 1
_____
70 + 3
_____



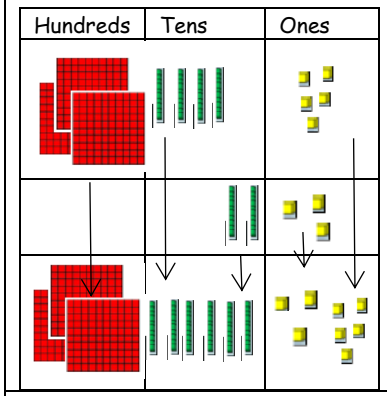
367 + 185 = 552

300 + 60 + 7
100 + 80 + 5
_____
500 + 50 + 2
_____
100 10





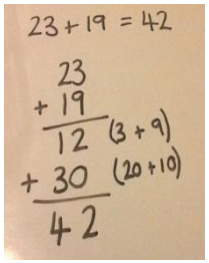
HTO + TO using base 10. No regrouping

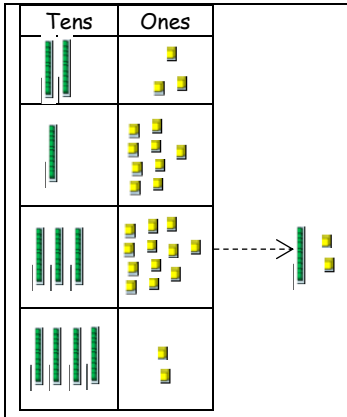


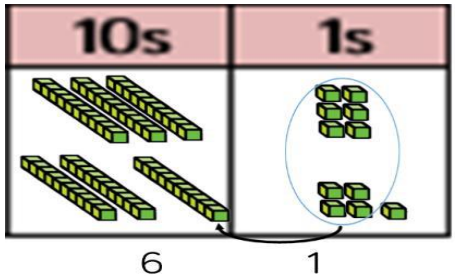
TO + TO regrouping using base 10.
Continue to develop understanding of partitioning and place value.

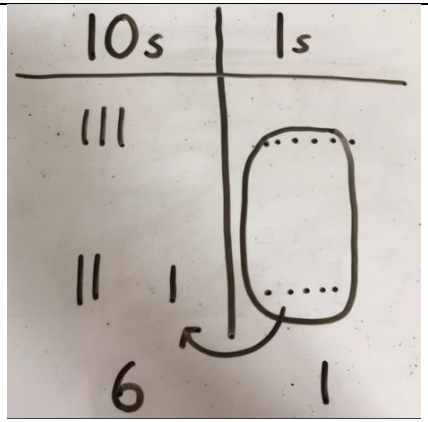
?	
21	34

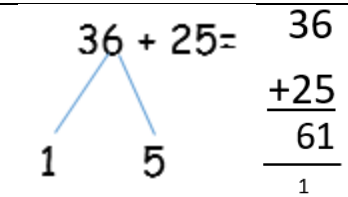
Looking for ways to make 10.
30 + 20 = 50 5 + 5 = 10
50 + 10 + 1 = 61
Formal method:

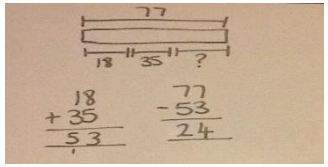


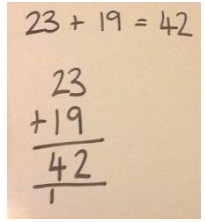

Tens
Ones

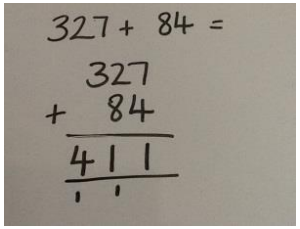

10s
1s
6
1

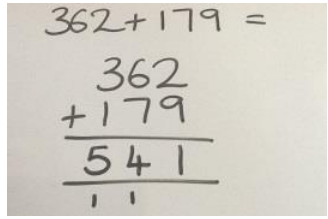

10s
1s
111
11
1
6
1

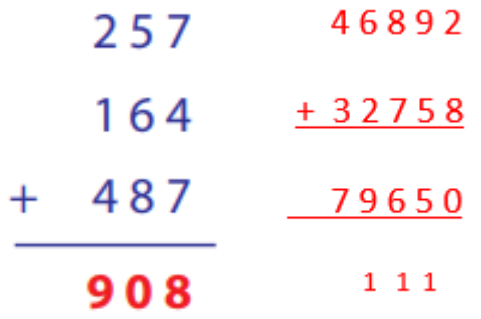

36 + 25=
1
5
36
+25
61
1


77
18
35
?
18
+35
53
77
-53
24


23 + 19 = 42
23
+19
42

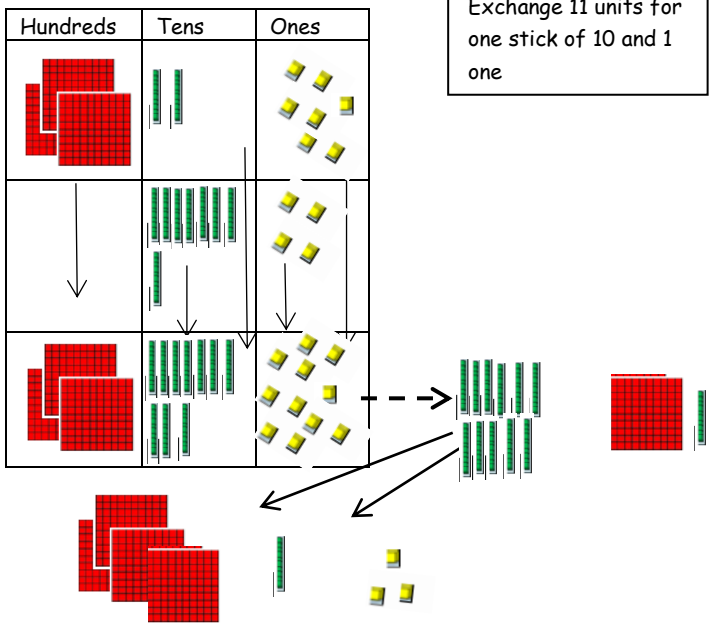

327 + 84 =
327
+ 84
411

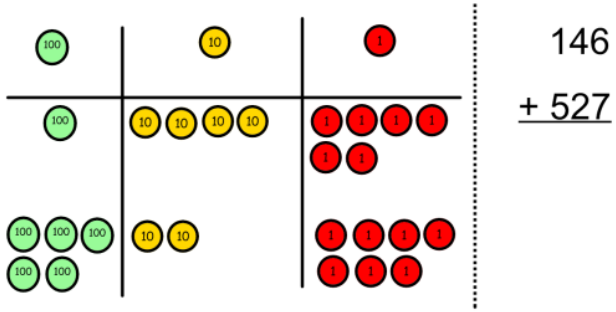

362+179 =
362
+179
541

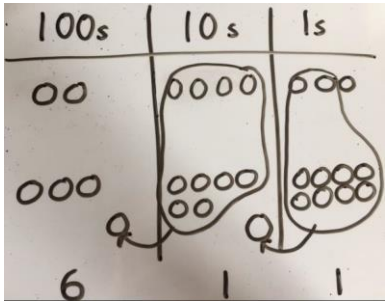

257
164
+ 487
908
46892
+ 32758
79650
1 1 1

HTO + TO, HTO + HTO etc. Regrouping

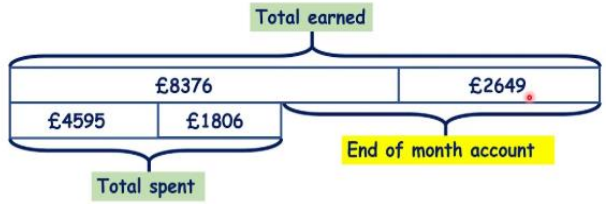
When there are 10 ones in the 1s column, we exchange for 1 ten. When there are 10 tens in the 10s column, we exchange for 1 hundred.



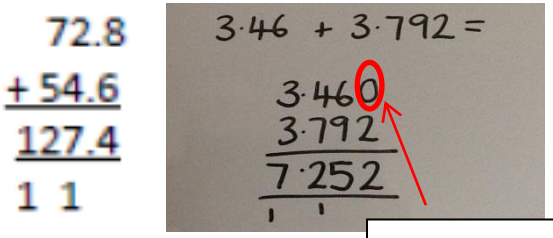




Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.



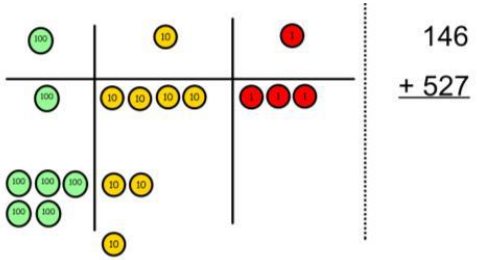
As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.



34.5 + 27.43 = 61.93

```
  34.50
+ 27.43
  61.93
   1
```

```
    £ 2 3 . 5 9
+   £   7 . 5 5
    £ 3 1 . 1 4
      1 1   1
```



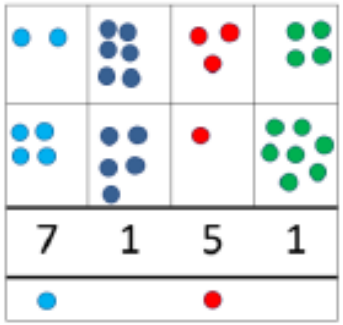
146

+ 527

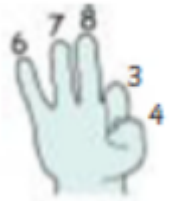
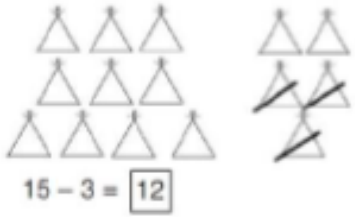
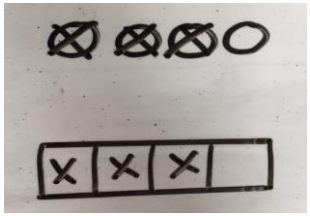
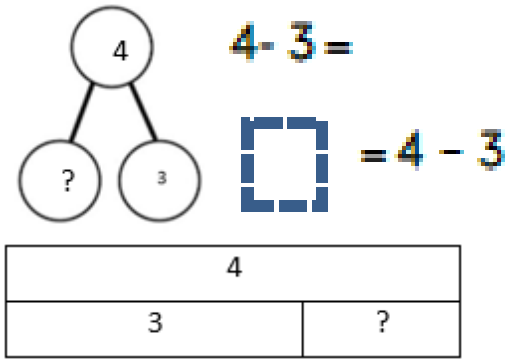
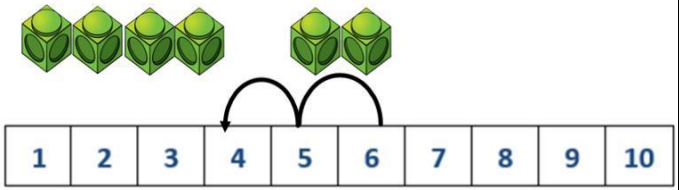
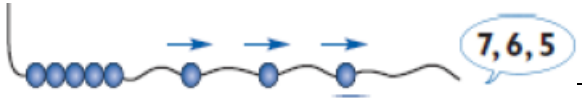
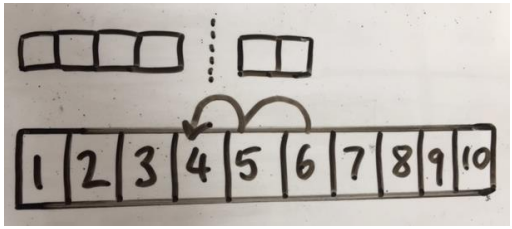
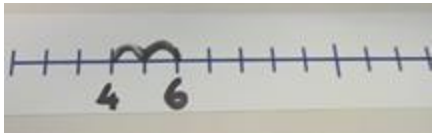
Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

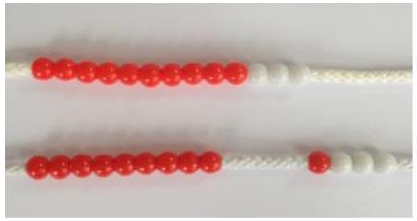
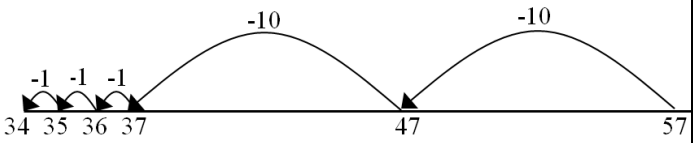
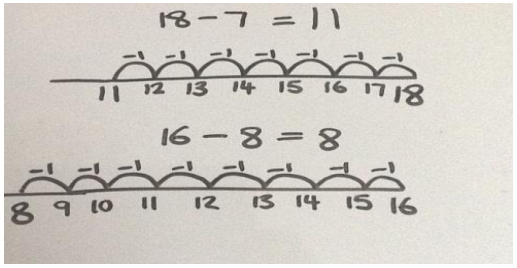
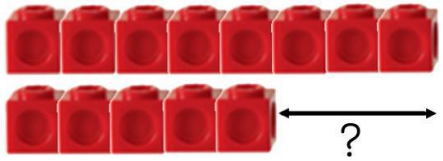
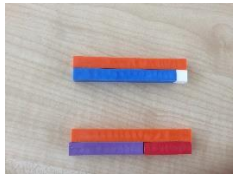
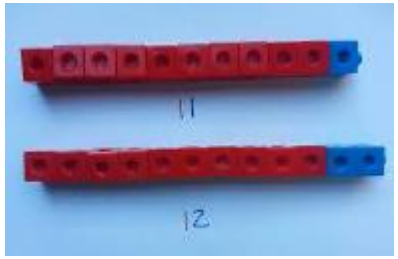
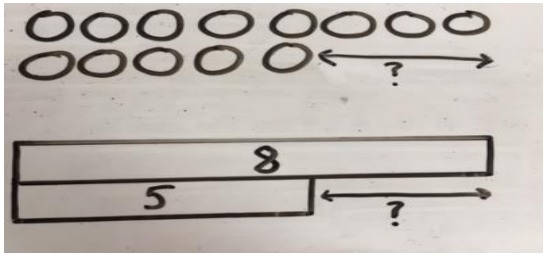
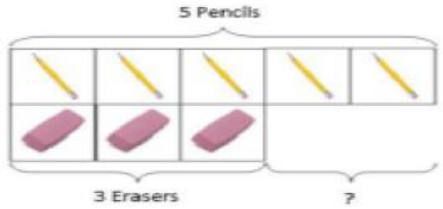
This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

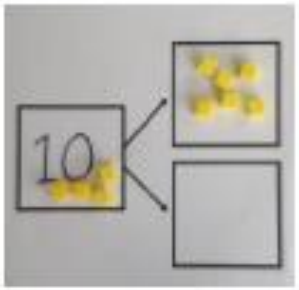
As children move on to decimals, money and decimal place value counters can be used to support learning.



Calculation policy - Subtraction
Key Language: take away, less than, the difference, subtract, subtrahend, minus, fewer, decrease, exchange, answer, inverse, count on, count back, reduction

Concrete	Pictorial	Abstract
Physically taking away and removing object from a whole (for example using counting objects, ten frames, cubes and other items such as beanbags).    6 – 2 = 4	 15 – 3 = 12 	**18 -3= 15** **8 – 2 = 6** 
Counting back (using number lines) children start with 6 and count back to 2.   	Children to represent what they see pictorially  	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line 

13 – 4 		 Put 13 in your head, count back 4. What number are you at?
Finding the difference (using cubes or Cuisenaire rods)   	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.  	Find the difference between 8 and 5. 8 – 5, the difference is ? Children to explore why 9 - 6 = 8 – 5 = 7 – 4 = have the same difference. 

<table>
<tr>
<td> 
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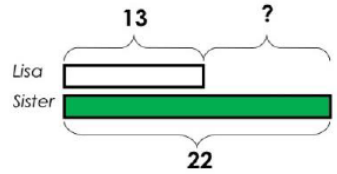
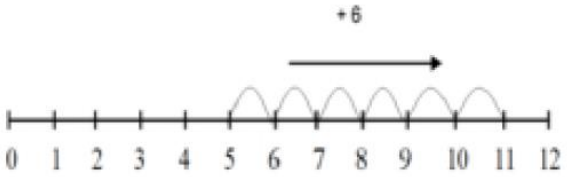
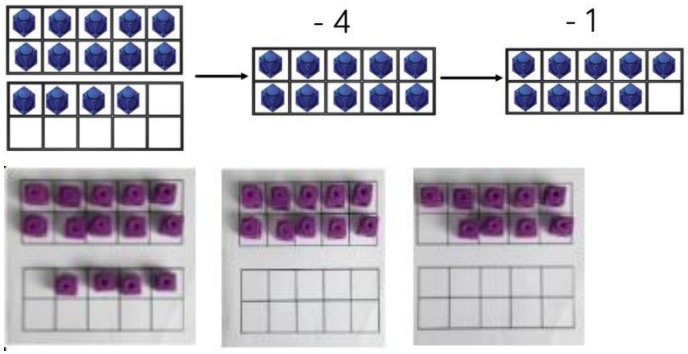
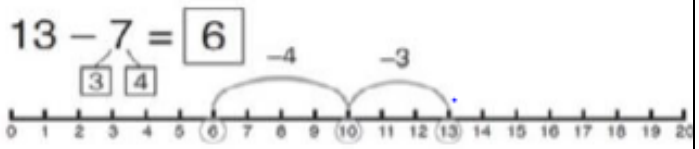
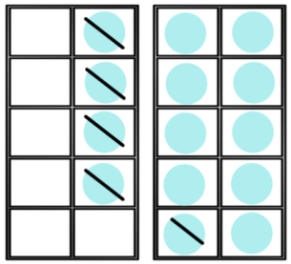
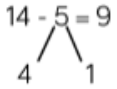
<td>Comparison bar models.
 

count on to find the difference
 
</td>
<td></td>
</tr>
<tr>
<td>Making 10 using ten frames.
 

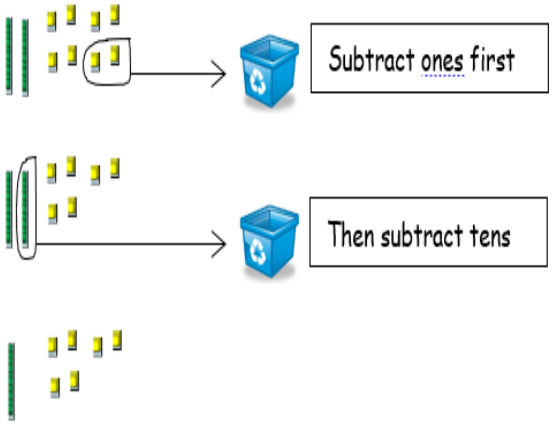
Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.</td>
<td> 

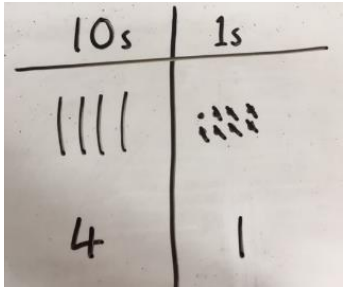
  </td>
<td>16 – 8=
How many do we take off to reach the next 10?
How many do we have left to take off?

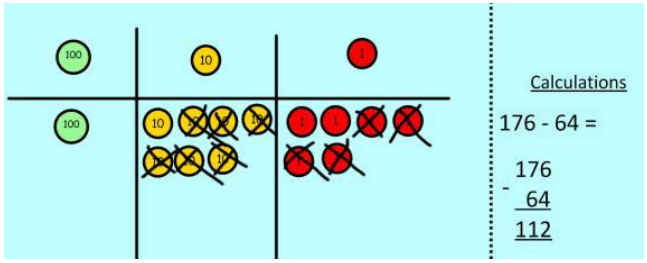
Children to show how they can make 10 by partitioning the subtrahend.
 

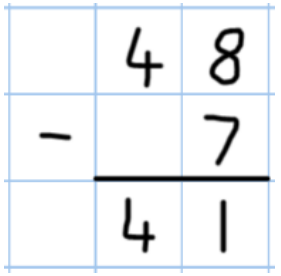
14 – 4 = 10
10 – 1 = 9</td>
</tr>
</table>

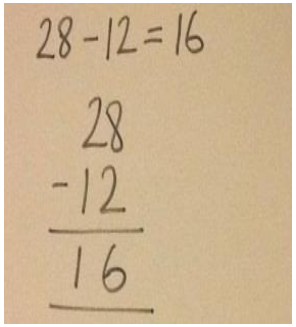
Formal column method - no exchange using base 10.



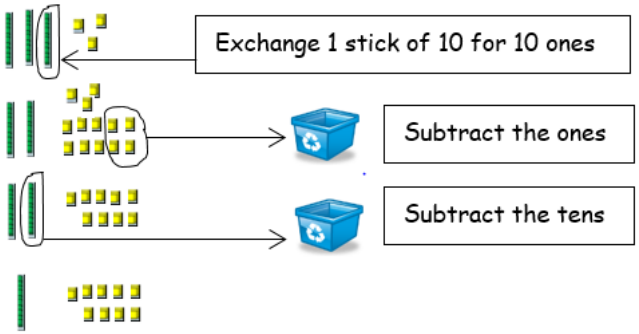




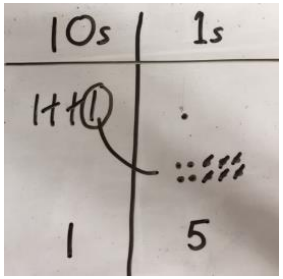




Formal column method - exchange.

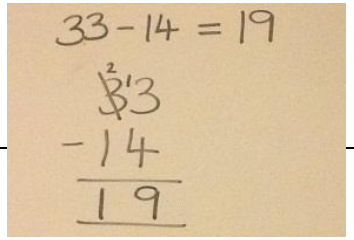


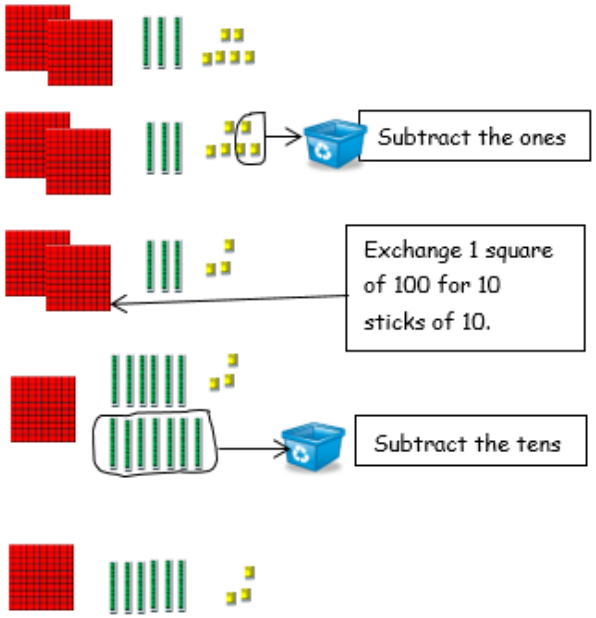
Represent the base 10 pictorially, remembering to show the exchange.



41 – 26 =

391	
186	?

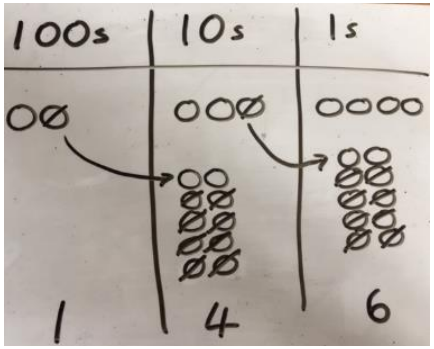


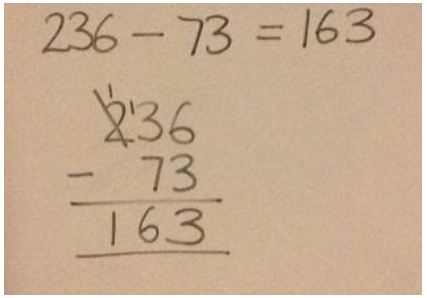


Formal column method using place value counters.

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

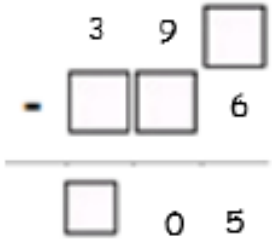
Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.



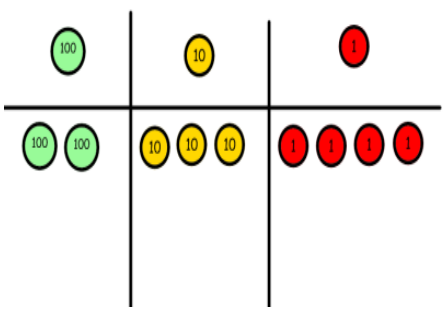


Children must understand what has happened when they have crossed out digits.

Solve the missing digits

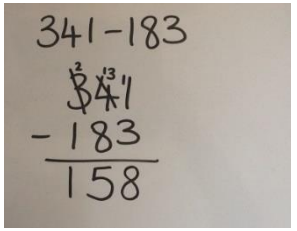


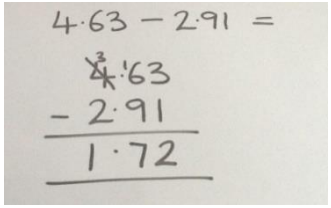
Make the larger number with the place value counters

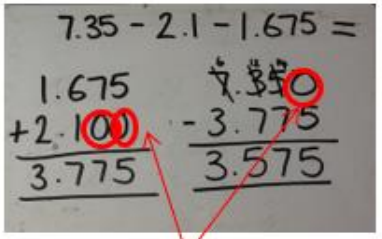


Calculations

234
- 88



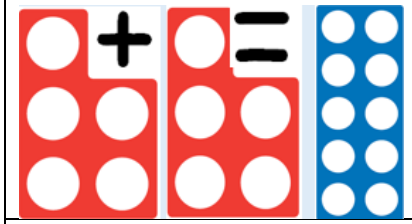
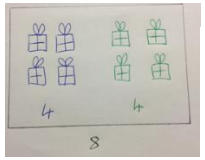
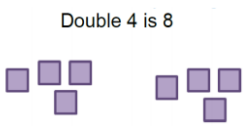
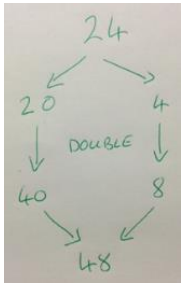
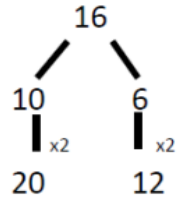
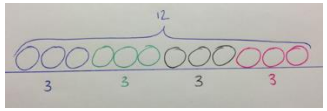


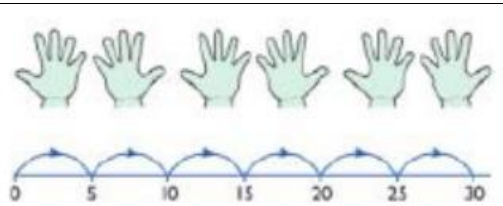


Calculation Policy: Multiplication and Division Guidance

	EYFS	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Multiplication	ELG: solve problems, including doubling	Doubling	Arrays – showing commutative multiplication	Arrays 2d x 1d	Column multiplication – introduced with place value counters (2 and 3 digit multiplied by 1 digit)	Column multiplication Abstract only but might need a repeat of year 4 first (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication Abstract methods
Division	ELG: solve problems, including halving and sharing	Sharing objects into groups	Division as grouping Division within arrays – linking to multiplication Repeated subtraction	Division with a remainder 2d divided by 1 d using base 10 or place value counters	Division with a remainder Short division (up to 3 digits by 1 digit – concrete and pictorial)	Short division (up to 4 digits by a 1 digit number including remainders)	Short division Long division with place value counters (up to 4 digits by a 2 digit) Children should exchange into the tenths and hundredths column too.

Calculation policy - Multiplication
Key Language: double, multiples, multiply, multiplied by, times, lots of, groups of, product, repeated addition, array, row, column

Concrete	Pictorial	Abstract
Doubling - using practical activities to show how to double a number.   	Draw pictures to show how to double a number.  	4 + 4 = 8 Double 4 is 8 Partition a number and then double each part before recombining it back together.  
Counting in multiples (groups) supported concrete objects in equal group.   	Use a number line or pictures to continue support in counting in multiples. 	Count multiples of a number aloud. Write sequences with multiples of numbers: 2, 4, 6, 8, 10, 12 10, 20, 30, 40 , 50 5, 10, 15, 20, 25, 30

<table>
<tr>
<td></td>
<td> 
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<td></td>
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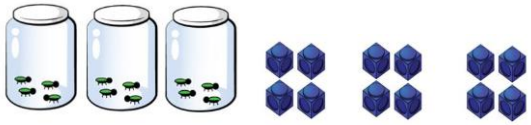
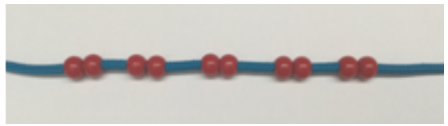
<tr>
<td>Repeated addition – use different objects to add equal groups
 
 
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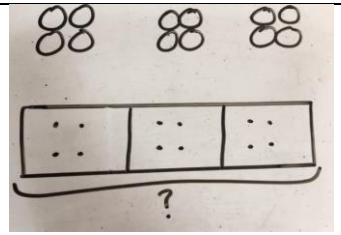
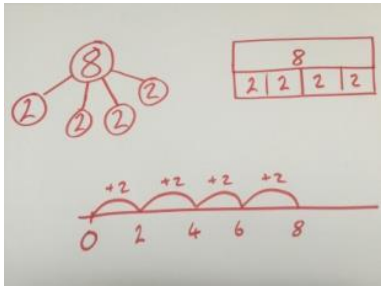
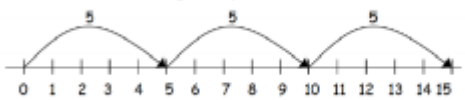
<td> 
 
 

5 + 5 + 5 = 15</td>
<td>There are 3 groups of 4 which is equal to 12

3 x 4 = 12 (3 groups of 4)

4 + 4 + 4 = 12

Write addition sentences to describe objects and pictures.

 
2 + 2 + 2 + 2 + 2 = 10</td>
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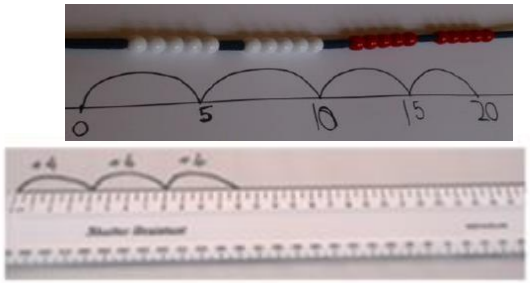
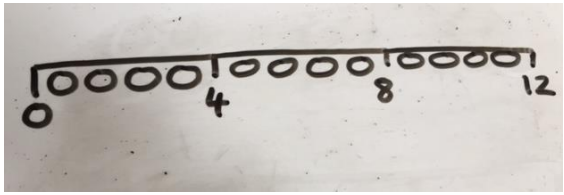
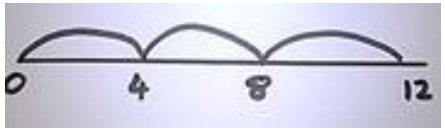
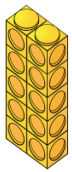
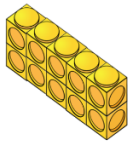
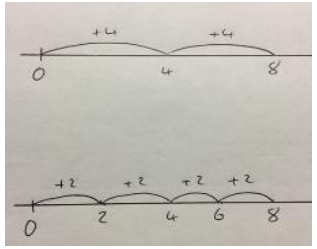
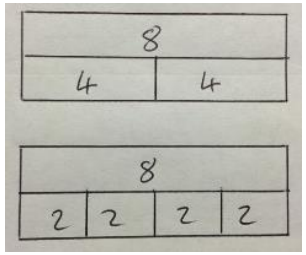
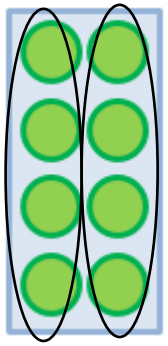
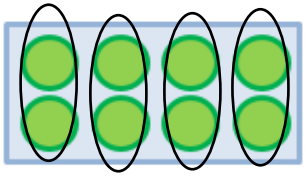
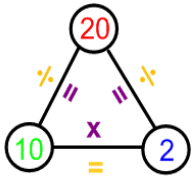
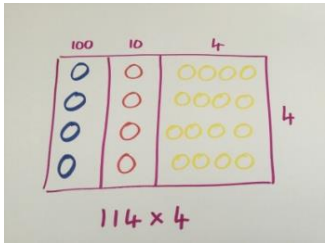
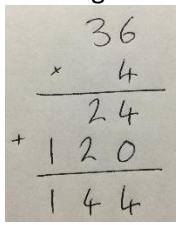
</table>

Number lines to show repeated groups-  	Represent this pictorially e.g. alongside a number line  	Abstract number line showing three jumps of four. 3 × 4 = 12 
Commutative – create arrays using counters/ cubes to show multiplication sentences.  2 lots of 5  5 lots of 2  	Number line showing 2x4 and 4x2 and bar model   Draw arrays in different rotations to find commutative multiplication sentences.	Use an array to write multiplication sentences and reinforce repeated addition 2 x 5 = 10 5 x 2 = 10 2 + 2 + 2 + 2 + 2 = 10 5 + 5 = 10  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

	 4 2 times 4 x 2 = 8  2 4 times 2 x 4 = 8  *Recall multiplication and division facts for 2, 5, 10*	
Partitioning to multiply as a stepping-stone to the formal method. Show the link using counters with arrays.	Children can represent the work they have done with place value counters in a way they understand. They can draw the counters, using colours to show different amounts. 	Start with multiplying by 1-digit and showing the clear addition. 

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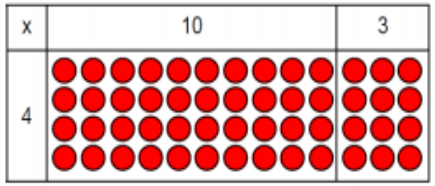
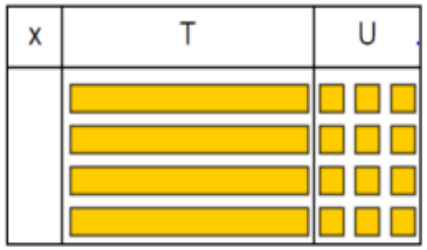


4 rows of 10
4 rows of 3

Move on to using Base 10 to move towards a more compact method.



4 rows of 13
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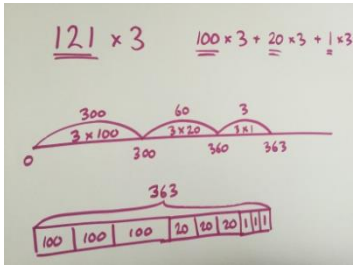
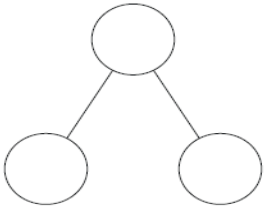
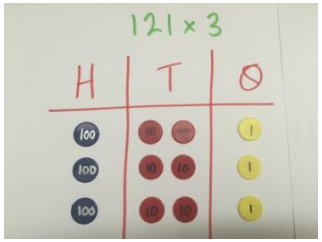
<td>



</td>
<td>
6 × 23 =

□ = 6 × 23

6 × 23

23 × 6

20 + 4
x 3
12
60
72
</td>
</tr>
<tr>
<td>
Column multiplication
Children can continue to be supported by place value counters.


</td>
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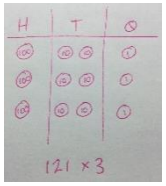
Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.

Children to represent the counters pictorially



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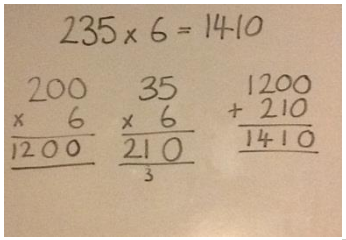

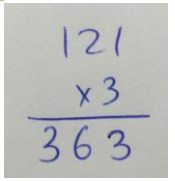

</td>
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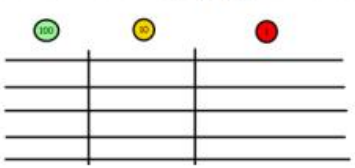
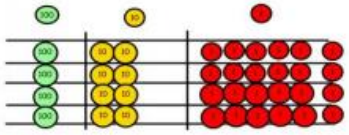
Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.



Calculations

4 x 126

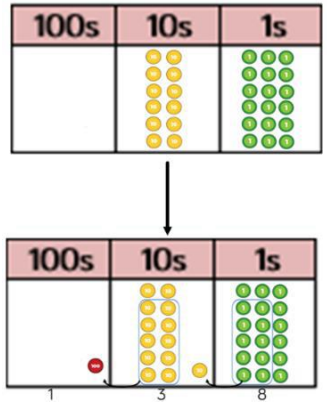
Fill each row with 126.

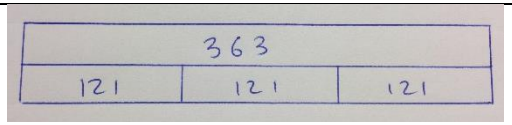


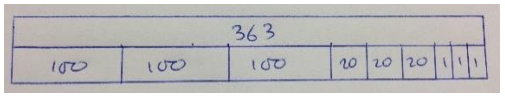
Calculations

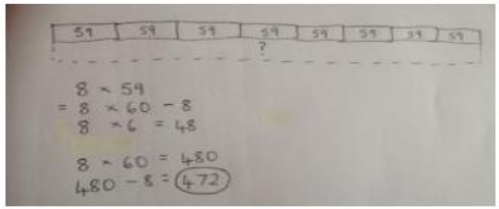
4 x 126

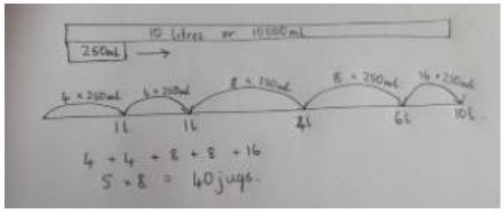
Add up each column, starting with the ones making any exchanges needed.

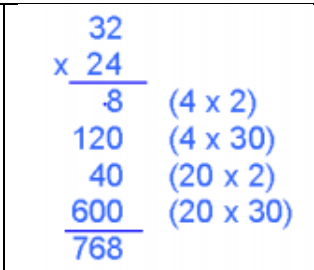


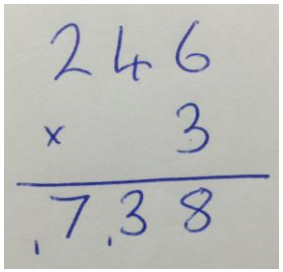


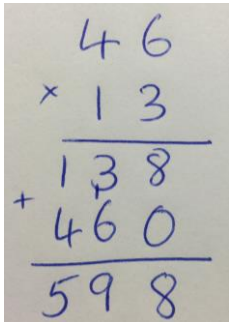


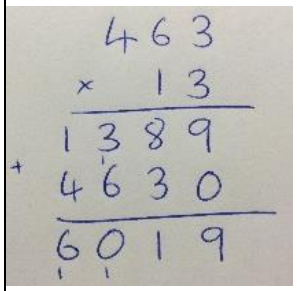


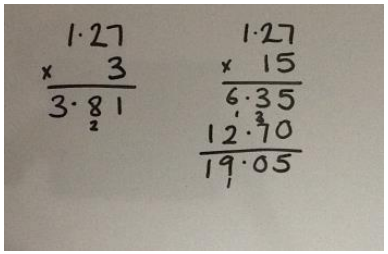




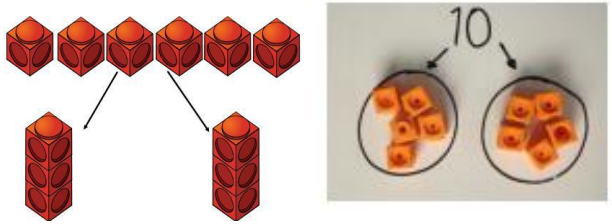


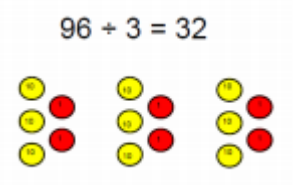
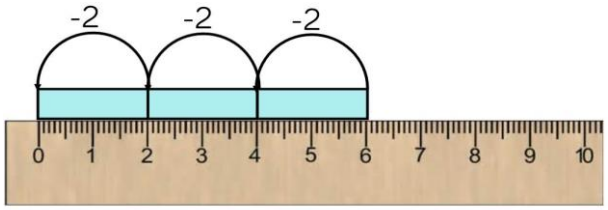
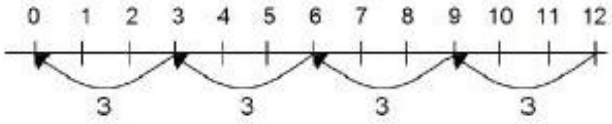
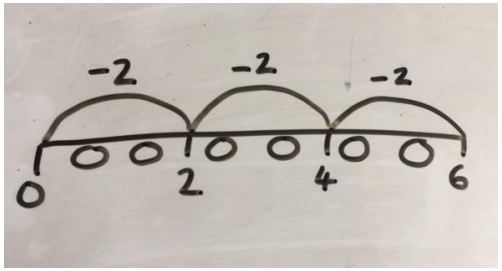
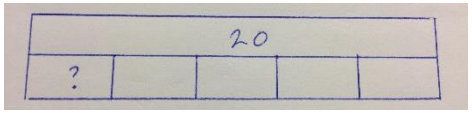


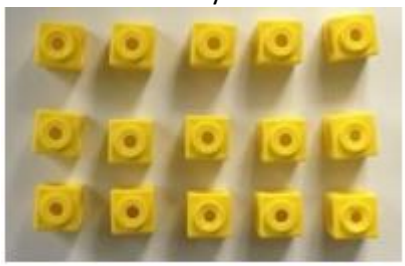
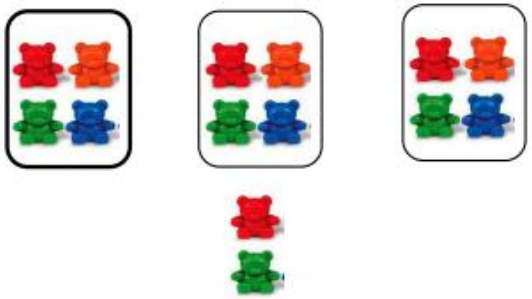
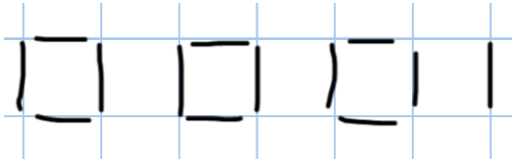




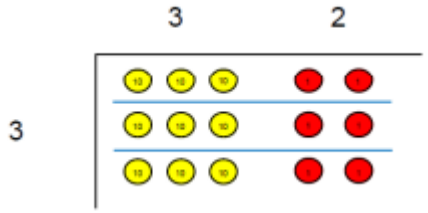
Calculation policy - Division
Key Language: halve, half, share, group, repeated subtraction, divide, divided by, divisor, dividend, quotient, into lots of, divisible, remainder, factor

Concrete	Pictorial	Abstract
Sharing using a range of objects The divisor will give you the number of groups.    	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4 Drawing objects that are shared	Share 9 buns between 3 people. 9 ÷ 3 = 3 3 | 3 6 ÷ 2 = 3 Children should also be encouraged to use their 2 times tables facts. Share 20 cubes: I can make ____ equal groups of 2. I can make ____ equal groups of 5. I can make ____ equal groups of 10. *What would happen if there were 21 cubes?* Share 8 cakes equally between 2 plates. *8 cakes shared equally between 2 is 4.*

		Share the 20 cubes equally between 5 friends. *20 shared between 5 equals 4.*
Grouping (repeated subtraction) Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding     3 groups of 2	Use a number line to show jumps in groups. The number of jumps equals the number of groups.   	 20 ÷ 5 = ? 5 x ? = 20 Find the inverse of multiplication and division sentences by creating four linking number sentences. **7 x 4 = 28** **4 x 7 = 28** **28 ÷ 7 = 4** **28 ÷ 4 = 7**

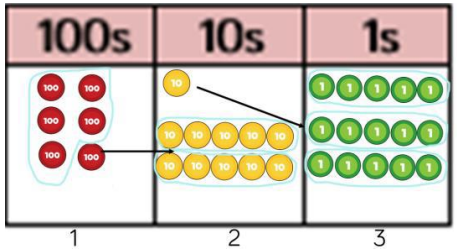
Division with arrays  15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 =15		
2digit ÷ 1digit with remainders  13÷4 = Use of lollipop sticks to form wholes – squares are made a dividing by 4  3 whole squares and 1 remainder	13 ÷ 4 = 3 remainder 1 	13 ÷ 4 = 3 r 1 13 ÷ 4 = 3.25 13 ÷ 4 = 3 ¼ 

Short division



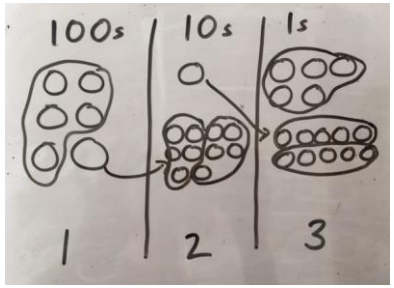
Use place value counters to divide using the bus stop method alongside

615÷5 = 123

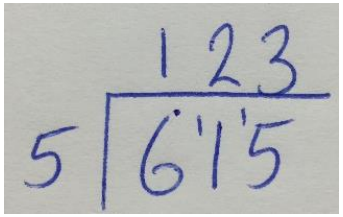


1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

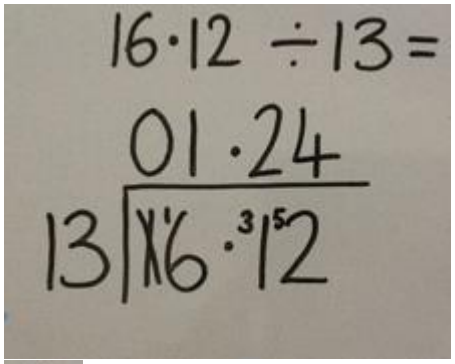
615÷5 = 123

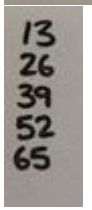


615÷5 = 123



5
10
15
20





Write the remainder as a fraction and/or decimal

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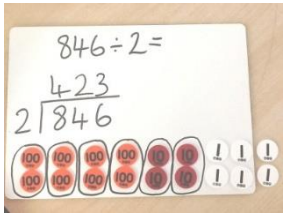
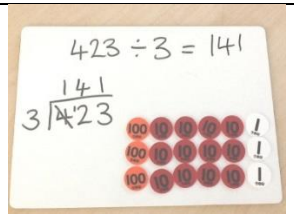
<td> 

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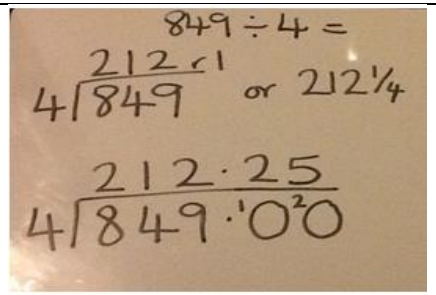
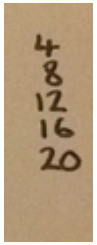
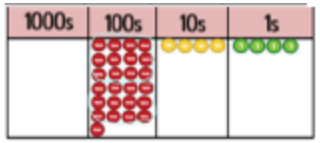
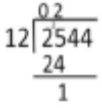
<td> 

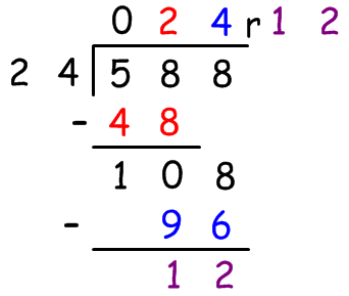

</td>
</tr>
<tr>
<td>Long division
2544÷12 =

we can’t group 2 thousands into 12s so we will exchange them.



We can group 24 hundreds into groups of 12 which leaves 1 hundred.</td>
<td></td>
<td> 
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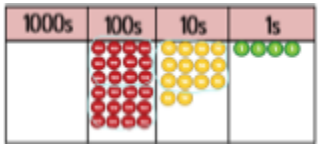
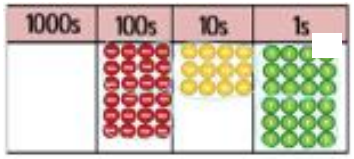


$$\begin{array}{r} 021 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$

After exchanging the hundred, we have 14 tens. We can group then into 12s and have 2 tens left over.



$$\begin{array}{r} 0212 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$

After exchanging the 2 tens, we have 24 ones. We can group them into 2 groups of 12 which leaves no remainders.